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Since $\frac{126}{27}\sqrt{-3} = \frac{3}{2}(a-b)(b-c)(a-c)\sqrt{-3}$, the factors are probably $\pm 2\sqrt{-3}$, $\pm\frac{3}{2}\sqrt{-3}$ and $\pm\frac{7}{2}\sqrt{-3}$. Hence

$$(1)\quad \alpha(\alpha^2 - 3\beta) = 55, \qquad (2)\quad \sqrt{-\beta}\,(3\alpha^2 - \beta) = 126\sqrt{-3};$$

$$\sqrt{-\beta} = 2\sqrt{-3};\quad 3\alpha^2 = 75,\quad \alpha = \pm 5;\quad \alpha = -5;$$

$$\sqrt{-\beta} = \frac{3}{2}\sqrt{-3};\quad 3\alpha^2 = \frac{363}{4},\quad \alpha = \pm\frac{11}{2};\quad \alpha = \frac{11}{2};$$

$$\sqrt{-\beta} = -\frac{7}{2}\sqrt{-3};\quad 3\alpha^2 = \frac{3}{4},\quad \alpha = \pm\frac{1}{2};\quad \alpha = -\frac{1}{2};$$

$$y = \frac{1}{3}(-5 + 2\sqrt{-3}) + \frac{1}{3}(-5 - 2\sqrt{-3}) = -\frac{10}{3},$$

$$y = \frac{1}{3}\left(\frac{11}{2} + \frac{3}{2}\sqrt{-3}\right) + \frac{1}{3}\left(\frac{11}{2} - \frac{3}{2}\sqrt{-3}\right) = \frac{11}{3},$$

$$y = \frac{1}{3}\left(-\frac{1}{2} - \frac{7}{2}\sqrt{-3}\right) + \frac{1}{3}\left(-\frac{1}{2} + \frac{7}{2}\sqrt{-3}\right) = -\frac{1}{3};$$

$$x = 2,\ 9,\ \text{and}\ 5.$$

V.

An Extension of Taylor's Theorem

BY J. C. GLASHAN, *Ottawa, Canada.*

$$f(x+a) - f(x) = \int_0^a da\,\frac{d}{dx}f(x+a),\quad \left(1 - \int_0^a da\,\frac{d}{dx}\right)f(x+a) = f(x),$$

$$\therefore f(x+a) = \left(1 - \int_0^a da\,\frac{d}{dx}\right)^{-1} f(x). \qquad \text{I.}$$

$$\left(1 - \int_0^a da\,\frac{d}{dx}\right)^{-1}\int_0^a da\,f(x+b) = \int_0^a da\left(1 - \int_0^a da\,\frac{d}{dx}\right)^{-1} f(x+b)$$

$$\text{(by I,)}\qquad = \int_0^a da\left\{1 - \int_0^{a+b} d(a+b)\,\frac{d}{dx}\right\}^{-1} f(x). \qquad \text{II.}$$

Expanding by I, but modifying each remainder by II before proceeding to obtain the term next following, we get

$$f(x+a+b+c+e+\&c.) = f(x+b+c+e+\&c.) + \int_0^a da\,f'(x+c+e+\&c.)$$
$$+ \int_0^a da\int_0^{a+b} d(a+b)\,f''(x+e+\&c.)$$
$$+ \int_0^a da\int_0^{a+b} d(a+b)\int_0^{a+b+c} d(a+b+c)\,f'''(x+\&c.) + \&c., \qquad \text{III.}$$

which is the extension proposed.

Writing ${}^{m}C$ for a function of $a_0, a_1, a_2, \ldots a_{m-1}$, and ${}^{n-m}\omega\,.{}^{m}C$ for the result of substituting, in ${}^{m}C$, $a_1 + a_2 + a_3 + \ldots a_{n-m}$ for a_0, a_{n-m+1} for a_1, a_{n-m+2} for a_2, &c., that is, if ${}^{m}C \equiv F(a_0, a_1, a_2, \ldots . a_{m-1})$, then

$$ {}^{n-m}\omega\,.{}^{m}C \equiv F(a_1 + a_2 + a_3 + \ldots a_{n-m}, a_{n-m+1}, a_{n-m+2}, \ldots a_{n-1}), $$

and determining these C-functions by

$$ {}^{1}C \equiv a_0, $$

$$ {}^{n}C \equiv a_0^n + \frac{n}{1} a_0^{n-1} ({}^{n-1}\omega\,.{}^{1}C) + \frac{n(n-1)}{1\,.\,2} a^{n-2} ({}^{n-2}\omega\,.{}^{2}C) + \ldots . + \frac{n}{1} a_0 ({}^{1}\omega\,.{}^{n-1}C), $$

III may be put under the form

$$ f(x) = f(x - a_0) + \frac{{}^{1}C}{1} f'(x - a_0 - a_1) + \frac{{}^{2}C}{1\,.\,2} f''(x - a_0 - a_1 - a_2) + \ldots . . $$

$$ \ldots . . + \frac{{}^{n}C}{n!} f^{n}(x - \Sigma_0^n a_r) + \&\text{c.} \qquad \text{IV.} $$

Writing a for a_0, then if $a_1 = a_2 = a_3 = \ldots = a_n = b$, this becomes the Series of Abel given by M. J. Bertrand in his *Traité de Calcul differentiel*, p. 324.

$$ f(x) = f(x - a) + \frac{a}{1} f(x - a - b) + \frac{a(a + 2b)}{1\,.\,2} f\{x - (a + 2b)\} + \ldots . $$

$$ \ldots . . + \frac{a(a + nb)^{n-1}}{n!} f\{x - (a + nb)\} + \&\text{c.} $$

If $b = 0$ this becomes Taylor's Theorem, or the simple expansion of I as evidently it should.

Using a, b, c, d, e, &c., instead of a_0, a_1, a_2, &c., we get for the first five coefficients of IV,

$$ {}^{1}C \equiv a, $$

$$ {}^{2}C \equiv a^2 + 2ab, $$

$$ {}^{3}C \equiv a^3 + 3a^2 (b + c) + 3a (b^2 + 2bc), $$

$$ {}^{4}C \equiv a^4 + 4a^3 (b + c + d) + 6a^2 \{(b + c)^2 + 2(b + c) d\} + 4a \{b^3 + 3b^2 (c + d) + 3b (c^2 + 2cd)\}, $$

$$ \begin{aligned} {}^{5}C \equiv a^5 &+ 5a^4 (b + c + d + e) + 10a^3 \{(b + c + d)^2 + 2(b + c + d) e\} \\ &+ 10a^2 \{(b + c)^3 + 3(b + c)^2 (d + e) + 3(b + c)(d^2 + 2de)\} \\ &+ 5a\,[b^4 + 4b^3 (c + d + e) + 6b^2 \{(c + d)^2 + 2(c + d) e\} \\ &+ 4b \{c^3 + 3c^2 (d + e) + 3c (d^2 + 2de)\}]. \end{aligned} $$